IMMC 2019 Greater China Problem C (Winter Season)

Calibrating Accelerometers

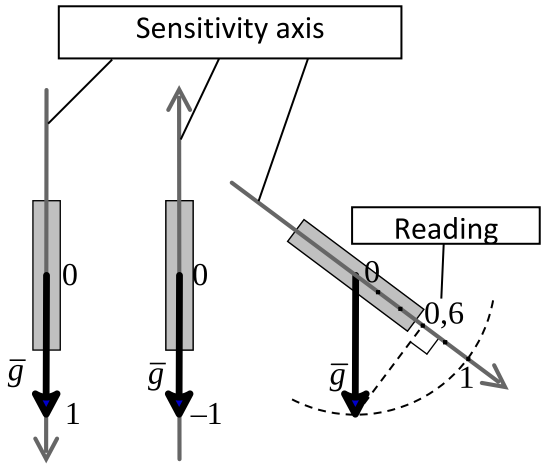

Figure C.1

Modern smart electronic devices can function as a navigator, or a pedometer, and can recognize whether their user walks or runs, goes by car or by bus; they can determine the orientation of the device in space and position the image on the screen accordingly. In solving all these and many other tasks, so-called *accelerometers* are used. The simplest (single-channel) accelerometer has a specific direction, its *sensitivity axis*. When this sensitivity axis is perfectly downwards vertically, a *motionless* accelerometer will record the reading of exactly 1g where g denotes the acceleration of gravity. In general, the readings of a motionless accelerometer allow one to calculate the deviation of its axis from downwards vertically, i.e. from the direction of gravity. If several single-channel accelerometers are attached to the device, then its orientation in space as a whole can be determined from the positions of the accelerometer axes.

However, it is impossible to make a perfect accelerometer, avoiding defects completely. Defects in sensors lead to errors in readings such as the systematic shift of all readings by a certain amount and their proportional change, i.e. increase or decrease of a certain number of times. Defects in mounting or manufacture of the sensors in the accelerometer case can lead to a slight deviation of their sensitivity axes from the axes of the case.

To find out exactly how a particular accelerometer distorts its readings, and make a digital correction of these distortions, the calibration procedure is carried out. One method of calibration is to take the accelerometer readings in several precisely fixed positions of its case and, using these data, create a formula that links the accelerometer readings subjected to the distortion with its position. This formula can then be used to determine the orientation of the accelerometer in an arbitrary position.

Your team's task is to suggest formulas/models that allow you to calculate the expected readings of real (having defects) accelerometers given the position of their case in the space. To create the formulas/models, use the provided calibration data sets.

The task has three versions with increasing difficulty depending on the number of single-channel sensors. Each of the versions contains an additional task, which can increase the overall assessment of your work if performed correctly, provided that the main task has been solved. In this task, you have to solve the inverse problem too, i.e. to create formulas and/or describe a method that allows one to calculate the spatial position of the case of an accelerometer set given their real readings.

In all versions, readings of the accelerometers are taken when they are *on a motionless fixed position*, with unit of measurement being g, the acceleration of gravity. The readings are given as a sequence of values measured at small equal intervals of time as their common case is set at given fixed positions and then turned from position to position (given in Excel files xC_f_eng). Given the imperfection and noise, even for the same position measurements will differ. On the other hand, the reading of an ideal (non-distorted) accelerometer directed vertically downwards should be equal to 1, and for the axis directed upwards, it will be –1; while for a tilted one, it should be equal to the projection of vector $\bar{g}$ onto its axis. The fixed positions used for the measurements are described by the projections of vector $\bar{g}$ onto the axes of ideal accelerometers in these positions, which are given in the Excel files xC_g_eng. (See **Figure C.1**.)

Version 1.0 Single-channel accelerometer

In this task, the sensitivity axis is assumed to coincide exactly with the axis of accelerometer's case. Its readings are measured in two positions:

a. the axis directed vertically downwards;
b. the axis directed vertically upwards.

Also, it is assumed that the position of the accelerometer changes only in one (vertical) plane, i.e. it is completely determined by the angle between its sensitivity axis and the downward direction.

Data: files 1C_f_eng.xls and 1C_g_eng.xls.

Version 2.0 Double-channel accelerometer

Here the accelerometer consists of two single-channel accelerometers X and Z, which are rigidly connected into a single unit attached to their common case so that their axes are directed along the X and Z axes of the case. However, due to mounting defects, they slightly deviate from these directions in the XZ plane. Their readings are measured in four positions of the case described in a separate file by the projections of the vector $\bar{g}$ on the case axes (in each position one of the X and Z axes is directed exactly up or down).

In this task, we assume that the position of the accelerometer changes only in its XZ plane, while its Y axis is horizontal and stays put.

Data: files 2C_f_eng.xls and 2C_g_eng.xls.

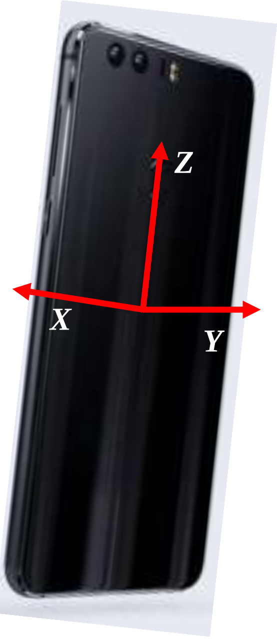

Figure C.2

Version 3.0 Three-channel accelerometer

The accelerometer consists of three single-channel ones, rigidly connected into a single unit set in the case so that the three sensitivity axes are directed along the X, Y, and Z axes of the case, but because of mounting defects, the two triples of axes do not coincide exactly. See **Figure C.2**. The accelerometer readings are measured in 20 positions described in a separate file.

In this task, there are no restrictions on the possible positions of the accelerometer.

Data: files 3C_f_eng.xls and 3C_g_eng.xls.

Note 1. If you have solved version 2 or 3 of the task, then it may not be necessary to solve the previous ones. Thus, having solved version 2, you may skip version 1, and having solved version 3, you may skip tasks 1 and 2.

Note 2. In the files xC_g_eng.xls, the lines with the projections of $\overline{g}$ on the axes of the case go in the same order as the case positions change during the measurements.

Hint. To understand how to extract necessary information from the measurement data, plot and study the graphs of accelerometer readings as a function of time.

Submission: Your solution paper should include a 1-page Summary Sheet. The body cannot exceed 20 pages for a maximum of 21 pages with the Summary Sheet inclusive. The appendices and references should appear at the end of the paper and do not count towards the 21 pages limit.

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