IMMC 2017 Region of Zhonghua Problem A

MEASUREMENTS ON EARTH VIA REMOTE SENSING TECHNIQUES

1. Background

Our Earth is home to all human beings. We are constantly seeking new knowledge about Earth, from its physical properties to its biodiversity. Historically measuring the shape, size and the surface variations of Earth has fascinated generations of scientists. With rapid advances in technology this endeavor has become increasingly important and feasible.

It is well-known that the shape of Earth, on the macro-scale, is approximately an ellipsoid. But on the scale of our daily life its surface is in fact rough and irregular. There are oceans, high mountains, plains, hills and deserts, etc. Accurately measuring the real shape and size on Earth is one of the major application directions of remote sensing technology.

2. Description

Based on Kepler's Laws, the orbits of satellites are elliptic with Earth. This elliptical orbit can be described using 6 orbital elements. For example, in Figure 1, we use a non-rotational equatorial earth-centered coordinate system XYZ as reference, the 6 orbital elements are: semi-major axis α, eccentricity e, inclination i, argument of periapsis ω, longitude of the ascending node Ω, the time of perihelion passage τ. In any given time t, the orbital satellite positions can be determined by these 6 parameters.

(1) **inclination** i: vertical tilt of the elliptical orbit of satellite with respect to the reference plane – Earth's equatorial plane.

(2) **semi-major axis** α: this is a parameter that confirms the size of the orbit. For circular orbits, it is the radius of the circle; for elliptical orbits, this is the semi-major axis.

(3) **eccentricity** e: this is a parameter that confirms the shape of the orbit. When $e = 0$, the orbit is a circle, when $0 < e < 1$ it is an ellipse.

(4) **longitude of the ascending node** Ω: this is a parameter to confirm the orbital plane, as an angle measured on the Equatorial Plane from the equinox to the

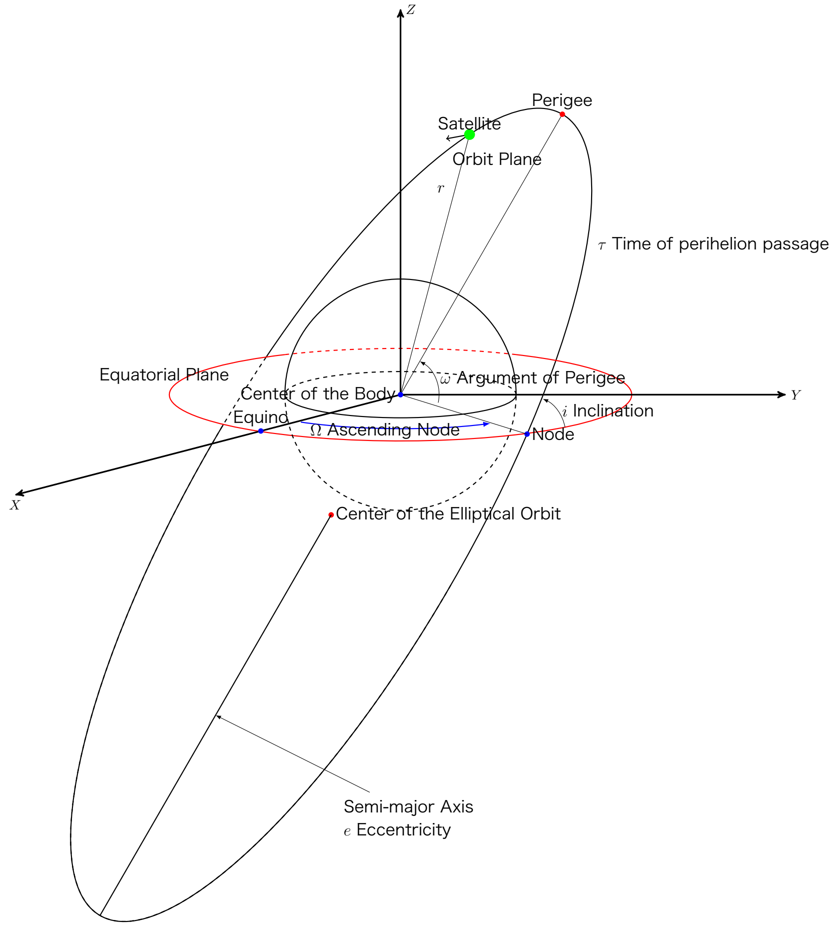

Figure 1. An Orbit of a Satellite

ascending node (the point where satellites will pass though on the Equatorial Plane from the Southern Hemisphere to the Northern Hemisphere)

(5) **time of perihelion passage** τ: the time where the satellite is passing through the closest point on the central object around which it orbits.

(6) **argument of periapsis** ω: defines the orientation of the ellipse in the orbital plane, as an angle measured from the ascending node to the periapsis (the closest point the satellite object comes to the primary object around which it orbits) and

this is also referred to as the angle between the line of ascending node and the line of perigee.

For more detailed introduction to orbital elements, please refer to:

`https://en.wikipedia.org/wiki/Orbital_elements`

Radar Altimeter (ALT) is an important sensor equipment for a remote sensing satellite. It can be used to measure the distance between the satellite itself and its projection point on Earth's surface (also known as the sub-satellite point (SSP)), and this is also the altitude of the satellite on Earth. The earlier results of Satellite Altimeter were to measure the shape of Earth and also the geoid, in order to calculate the global gravitational field. Through the wide coverage use of the data retrieved by the satellite altimeter, research on areas such as oceanography, geodesy, geophysics, climatology, hydrogeology and marine biology, etc., benefit enormously. Especially in the case of oceanology research, satellite altimeter provides a strong and powerful tool for research on the global oceanic surface, oceanic currents, and also the time-dependent changes associated with those data variables. Chinese Tiangong-2 space station and many more remote sensing satellites also equipped with this kind of equipment.

The basic principles of the Satellite Radar Altimeter are to transmit radio wave impulses to the sub-satellite point and measure the time lapse of the echo signal from the reflected surface on Earth, in order to measure the distance between the satellite and the reflected surface. If the time lapse is ΔT, then the height is $H = c \cdot \Delta T/2$, where c is the speed of light. As shown in Figure 2, due to the fact that the size of the antenna is relatively small, its radio signals transmitted propagates downwards along the conical space. The angle of cone θ is usually $1^\circ \sim 2^\circ$, and the illuminated zone on Earth's surface can been approximated as a circular region (also known as Footprint). The measurement of the altimeter in fact is the distance between the satellite and the sub-satellite point. For simplicity's sake, we can take the distance between all the points on this circular region and the satellite to be the same. In addition, Radar Altimeter repeatedly emits impulses in a very rapid way, and the neighboring footprint regions are just in close proximity. The union of these circular footprint regions become a belt shaped observation zone.

For more detailed introduction on radar altimeter, please refer to

https://en.wikipedia.org/wiki/Radar_altimeter and
https://en.wikipedia.org/wiki/Altimeter#Satellites

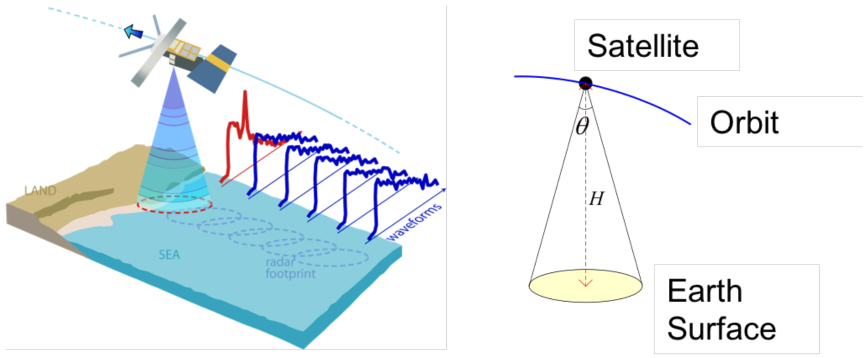

FIGURE 2. Theory Behind a Radar Altimeter

3. QUESTIONS

(1) Based on a simple mathematical model of a satellite orbit, please construct a mathematical model using the measurement data from a radar altimeter to calculate the shape of Earth's surface.

(2) Based on a satellite whose orbital elements and angle of cone (or size of the footprint region) are fixed, construct a mathematical model to analyze the minimum time needed to measure Earth once, as complete and even as possible. Select a remote sensing satellite in orbit, input the orbital elements into your constructed mathematical model, and calculate the time needed for that satellite to completely measure the whole Earth.

(3) If multiple altimeters are installed on a satellite, how can that satellite to make use of measurement data by these altimeters to determine its own spatial position?